RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4723/01

MATHEMATICS

Core Mathematics 3

MONDAY 11 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 Differentiate each of the following with respect to x.

(i) $x^3(x+1)^5$ [2]

(ii) $\sqrt{3x^4+1}$ [3]

2 Solve the inequality $|4x-3| < |2x+1|$. [5]

3 The function f is defined for all non-negative values of x by

$$f(x) = 3 + \sqrt{x}.$$

(i) Evaluate ff(169). [2]

(ii) Find an expression for $f^{-1}(x)$ in terms of x. [2]

(iii) On a single diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, indicating how the two graphs are related. [3]

4 The integral I is defined by

$$I = \int_0^{13} (2x+1)^{\frac{1}{3}}\,dx.$$

(i) Use integration to find the exact value of I. [4]

(ii) Use Simpson's rule with two strips to find an approximate value for I. Give your answer correct to 3 significant figures. [3]

5 A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula

$$m = 240e^{-0.04t}.$$

(i) Find the time taken for the substance to halve its mass. [3]

(ii) Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year. [4]

6 **(i)** Given that $\int_0^a (6e^{2x} + x)\,dx = 42$, show that $a = \frac{1}{2}\ln\left(15 - \frac{1}{6}a^2\right)$. [5]

(ii) Use an iterative formula, based on the equation in part **(i)**, to find the value of a correct to 3 decimal places. Use a starting value of 1 and show the result of each iteration. [4]

7 (i) Sketch the graph of $y = \sec x$ for $0 \leqslant x \leqslant 2\pi$. [2]

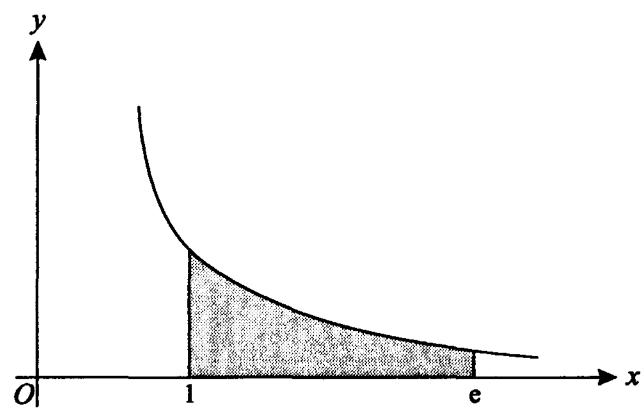
(ii) Solve the equation $\sec x = 3$ for $0 \leqslant x \leqslant 2\pi$, giving the roots correct to 3 significant figures. [3]

(iii) Solve the equation $\sec\theta = 5\operatorname{cosec}\theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving the roots correct to 3 significant figures. [4]

8 (i) Given that $y = \dfrac{4\ln x - 3}{4\ln x + 3}$, show that $\dfrac{dy}{dx} = \dfrac{24}{x(4\ln x + 3)^2}$. [3]

(ii) Find the exact value of the gradient of the curve $y = \dfrac{4\ln x - 3}{4\ln x + 3}$ at the point where it crosses the x-axis. [4]

(iii)



The diagram shows part of the curve with equation

$$y = \frac{2}{x^{\frac{1}{2}}(4\ln x + 3)}.$$

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = \mathrm{e}$ and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis. [4]

9 (i) Prove the identity

$$\tan(\theta + 60°)\tan(\theta - 60°) \equiv \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}. \quad [4]$$

(ii) Solve, for $0° < \theta < 180°$, the equation

$$\tan(\theta + 60°)\tan(\theta - 60°) = 4\sec^2\theta - 3,$$

giving your answers correct to the nearest $0.1°$. [5]

(iii) Show that, for all values of the constant k, the equation

$$\tan(\theta + 60°)\tan(\theta - 60°) = k^2$$

has two roots in the interval $0° < \theta < 180°$. [3]

1	**(i)**	Attempt use of product rule	M1		
		Obtain $3x^2(x+1)^5 + 5x^3(x+1)^4$	A1	**2**	or equiv
		[Or: (following complete expansion and differentiation term by term)			
		Obtain $8x^7 + 35x^6 + 60x^5 + 50x^4 + 20x^3 + 3x^2$	B2		allow B1 if one term incorrect]
	(ii)	Obtain derivative of form $kx^3(3x^4+1)^n$	M1		any constants k and n
		Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$	M1		
		Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$	A1	**3**	or (unsimplified) equiv
2		Identify critical value $x = 2$	B1		
		Attempt process for determining both critical values	M1		
		Obtain $\frac{1}{3}$ and 2	A1		
		Attempt process for solving inequality	M1		table, sketch …; implied by plausible answer
		Obtain $\frac{1}{3} < x < 2$	A1	**5**	
3	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (16 and hence) 7	A1	**2**	
	(ii)	Attempt correct process for finding inverse	M1		maybe in terms of y so far
		Obtain $(x-3)^2$	A1	**2**	or equiv; in terms of x, not y
	(iii)	Sketch (more or less) correct $y = \mathrm{f}(x)$	B1		with 3 indicated or clearly implied on y-axis, correct curvature, no maximum point
		Sketch (more or less) correct $y = \mathrm{f}^{-1}(x)$	B1		right hand half of parabola only
		State reflection in line $y = x$	B1	**3**	or (explicit) equiv; independent of earlier marks
4	**(i)**	Obtain integral of form $k(2x+1)^{\frac{4}{3}}$	M1		or equiv using substitution; any constant k
		Obtain correct $\frac{3}{8}(2x+1)^{\frac{4}{3}}$	A1		or equiv
		Substitute limits in expression of form $(2x+1)^n$ and subtract the correct way round	M1		using adjusted limits if subn used
		Obtain 30	A1	**4**	
	(ii)	Attempt evaluation of $k(y_0 + 4y_1 + y_2)$	M1		any constant k
		Identify k as $\frac{1}{3}\times 6.5$	A1		
		Obtain 29.6	A1	**3**	or greater accuracy (29.554566…)
		[SR: (using Simpson's rule with 4 strips)			
		Obtain $\frac{1}{3}\times 3.25(1 + 4\times\sqrt[3]{7.5} + 2\times\sqrt[3]{14} + 4\times\sqrt[3]{20.5} + 3)$ and hence 29.9	B1		or greater accuracy (29.897…)]

5	**(i)**	State $e^{-0.04t} = 0.5$	B1		or equiv
		Attempt solution of equation of form $e^{-0.04t} = k$	M1		using sound process; maybe implied
		Obtain 17	A1	**3**	or greater accuracy (17.328…)
	(ii)	Differentiate to obtain form $k\,e^{-0.04t}$	*M1		constant k different from 240
		Obtain $(\pm)\ 9.6e^{-0.04t}$	A1		or (unsimplified) equiv
		Equate attempt at first derivative to $(\pm)$ 2.1 and attempt solution	M1		dep *M; method maybe implied
		Obtain 38	A1	**4**	or greater accuracy (37.9956…)
6	**(i)**	Obtain integral of form $k_1e^{2x} + k_2x^2$	M1		any non-zero constants k_1, k_2
		Obtain correct $3e^{2x} + \frac{1}{2}x^2$	A1		
		Obtain $3e^{2a} + \frac{1}{2}a^2 - 3$	A1		
		Equate definite integral to 42 and attempt rearrangement	M1		using sound processes
		Confirm $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$	A1	**5**	AG; necessary detail required
	(ii)	Obtain correct first iterate 1.348…	B1		
		Attempt correct process to find at least 2 iterates	M1		
		Obtain at least 3 correct iterates	A1		
		Obtain 1.344	A1	**4**	answer required to exactly 3 d.p.; allow recovery after error
		$[1 \to 1.34844 \to 1.34382 \to 1.34389]$			
7	**(i)**	Show correct general shape (alternating above and below x-axis)	M1		with no branch reaching x-axis
		Draw (more or less) correct sketch	A1	**2**	with at least one of 1 and –1 indicated or clearly implied
	(ii)	Attempt solution of $\cos x = \frac{1}{3}$	M1		maybe implied; or equiv
		Obtain 1.23 or 0.392π	A1		or greater accuracy
		Obtain 5.05 or 1.61π	A1	**3**	or greater accuracy and no others within $0 \le x \le 2\pi$; penalise answer(s) to 2sf only once
	(iii)	Either: Obtain equation of form $\tan\theta = k$ M1			any constant k; maybe implied
		Obtain $\tan\theta = 5$	A1		
		Obtain two values only of form $\theta,\ \theta + \pi$	M1		within $0 \le x \le 2\pi$; allow degrees at this stage
		Obtain 1.37 and 4.51 (or 0.437π and 1.44π)	A1	**4**	allow ±1 in third sig fig; or greater accuracy
		Or: (for methods which involve squaring,etc.) Attempt to obtain eqn in one trig ratio	M1		
		Obtain correct value	A1		$\tan^2\theta = 25, \cos^2\theta = \frac{1}{26}$, …
		Attempt solution at least to find one value in first quadrant and one value in third	M1		
		Obtain 1.37 and 4.51 (or equivs as above)	A1		ignoring values in second and fourth quadrants

8	**(i)**	Attempt use of quotient rule	M1		allow for numerator 'wrong way round'; or equiv
		Obtain $\dfrac{(4\ln x+3)\frac{4}{x}-(4\ln x-3)\frac{4}{x}}{(4\ln x+3)^2}$	A1		or equiv
		Confirm $\dfrac{24}{x(4\ln x+3)^2}$	A1	**3**	AG; necessary detail required
	(ii)	Identify $\ln x = \frac{3}{4}$	B1		or equiv
		State or imply $x = e^{\frac{3}{4}}$	B1		
		Substitute e^k completely in expression for derivative	M1		and deal with $\ln e^k$ term
		Obtain $\frac{2}{3}e^{-\frac{3}{4}}$	A1	**4**	or exact (single term) equiv
	(iii)	State or imply $\int \dfrac{4\pi}{x(4\ln x+3)^2}\,dx$	B1		
		Obtain integral of form $k\dfrac{4\ln x-3}{4\ln x+3}$ or $k(4\ln x+3)^{-1}$	*M1		any constant k
		Substitute both limits and subtract right way round	M1		dep *M
		Obtain $\frac{4}{21}\pi$	A1	**4**	or exact equiv
9	**(i)**	Attempt use of either of $\tan(A \pm B)$ identities	M1		
		Substitute $\tan 60° = \sqrt{3}$ or $\tan^2 60° = 3$	B1		
		Obtain $\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta} \times \dfrac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}$	A1		or equiv (perhaps with tan 60° still involved)
		Obtain $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}$	A1	**4**	AG
	(ii)	Use $\sec^2\theta = 1+\tan^2\theta$	B1		
		Attempt rearrangement and simplification of equation involving $\tan^2\theta$	M1		or equiv involving $\sec\theta$
		Obtain $\tan^4\theta = \frac{1}{3}$	A1		or equiv $\sec^2\theta = 1.57735...$
		Obtain 37.2	A1		or greater accuracy
		Obtain 142.8	A1	**5**	or greater accuracy; and no others between 0 and 180
	(iii)	Attempt rearrangement of $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta} = k^2$ to form $\tan^2\theta = \dfrac{f(k)}{g(k)}$	M1		
		Obtain $\tan^2\theta = \dfrac{k^2+3}{1+3k^2}$	A1		
		Observe that RHS is positive for all k, giving one value in each quadrant	A1	**3**	or convincing equiv